

Comparison between the Fourier and Wavelet methods of spectral analysis applied to stationary and nonstationary heart period data

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Abstract

The aim of this study was to assess the error made by violating the assumption of stationarity when using Fourier analysis for spectral decomposition of heart period power. A comparison was made between using Fourier and Wavelet analysis (the latter being a relatively new method without the assumption of stationarity). Both methods were compared separately for stationary and nonstationary segments. An ambulatory device was used to measure the heart period data of 40 young and healthy participants during a psychological stress task and during periods of rest. Surprisingly small differences (<1%) were found between the results of both methods, with differences being slightly larger for the nonstationary segments. It is concluded that both methods perform almost identically for computation of heart period power values. Thus, the Wavelet method is only superior for analyzing heart period data when additional analyses in the time-frequency domain are required.

Descriptors: Wavelet, Fourier, Stationary, Spectral, Heart period

Heart rate (or heart period) variability has been found to be useful to assess autonomic nervous system activity. Results using spectral analysis of heart period variability have demonstrated that heart period variability power can be divided into three major frequency bands, which provide separate information concerning the sympathetic and parasympathetic nervous systems (see Berntson et al., 1997). The high frequency (HF) power (i.e., the respiratory frequency band) is considered to range (nominally) from about 0.15 Hz to 0.4 Hz. The integrated power within this high frequency band is generally believed to be an index of cardiac vagal control. The low frequency (LF) band generally ranges from 0.05 Hz to 0.15 Hz. The integrated power within this low frequency band is believed to reflect both sympathetic and vagal control of heart rate. Frequencies below 0.05 Hz are sometimes identified as very low and/or ultralow frequency (VLF, ULF) bands. Some researches also use the LF/HF power ratio as an index of the cardiac sympathovagal balance.

Recently, Berntson et al. (1997) published an extensive review article in which the origins, methods, and interpretative caveats of heart period variability measures were discussed in great detail. In summary, these authors concluded that patterns of heart period variability have proved useful in psychophysiological applications in the past and hold considerable promise as

psychophysiological measures in the future, but that there are methodological and interpretative pitfalls. One of the methodological pitfalls mentioned by Berntson et al. that is of particular relevance here is the assumption of stationarity. Stationarity of time series may be interpreted as having at least a stable mean and variance over time. Most widely used spectral analysis methods of heart period variability (like Fourier transformation) assume that the data show at least weak stationarity (i.e., have a stable mean and variance). However, it is difficult to find a clear estimate in the literature of the size of the error made as a result of violating this assumption. Especially in the fields of physiology of emotion and ambulatory measurements of heart period data, the assumption of stationarity might be strongly violated. The aim of this study is to assess the differences between Fourier transformation (a widely used stationarity-assumption-based method) and spectral analysis by the Wavelet method (a relatively new method that is not based on the assumption of stationarity) applied to the same interbeat interval (IBI) data set.

Heart period (uniformly spaced) time series may be transformed into a spectral density function by discrete Fourier transformation (DFT). After DFT, the integrated spectral density within a certain frequency window represents the power (or variance) of the signal for that specific frequency window. However, heart period time series may be transformed into power spectra by other techniques as well. Discrete Wavelet transformation (DWT) is a relatively new technique and has been an important topic in mathematics, science, engineering, and (more recently) economics. An extensive introduction to Wavelets and their applications can be

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found in Chui (1992). Usage of DWT for analyzing heart period variability has been described by Akay, Landesberg, Welkowitz, Akay, and Sapoznikov (1993), Pichot et al. (1999), and Wiklund, Akay, and Niklasson (1997). Wavelets allow simultaneous decomposition of a time series into components that are localized in both time and frequency. This is unlike Fourier transformation, where the obtained components are localized in the frequency domain only, and all time domain information is lost. By multiple convolution steps with a predetermined Wavelet base function, a time series can be transformed into several new levels of time series, with each level representing frequencies within a specified frequency window. The widths of the frequency windows and the number of points representing the original time area both vary with each convolution step. For DWT, an input signal of 2^n data points will be transformed with n convolution steps into a constant and $n - 1$ time series. The higher the frequency, the more Wavelet coefficients needed for that level to represent the signal within the original time interval. After DWT, the power can be calculated for each frequency window by summing the squares of the coefficients at that level. The selection of the Wavelet base that should be used for a data set is important because the DWT coefficients are a function of both the original data and the Wavelet base. Therefore, the selected base should "match" the original data set. To summarize, DWT may be used to calculate power values for specific frequency bands (analogous to DFT), without any assumption of stationarity, and it also offers some additional information that may be used to obtain an impression of how these power values fluctuate over time. Formulas of the DWT decomposition and reconstruction algorithms are shown in the Appendix.

Pichot et al. (1999) analyzed the same heart rate variability data both with the DFT and the DWT methods. Fourier and Wavelet transforms were computed from sequences of heart period intervals of six participants receiving increasing doses of atropine and propranolol. Their results demonstrated that power values obtained from the Wavelet transform gave a significantly better quantitative analysis of heart period variability than did power values obtained from the Fourier transform. Differences between the doses were more pronounced after Wavelet transformation.

We cannot conclude from the results of Pichot et al. (1999) whether differences between power values obtained by both methods are due to violations of the assumption of stationarity when the Fourier method is used, because they did not test their IBI data for stationarity. The Fourier method of spectral analysis assumes that the data show at least weak stationarity. The issue of nonstationarity has been discussed in detail elsewhere by several authors (Berntson et al., 1997; Grossman, 1992; Porges & Bohrer, 1990; Weber, Molenaar, & van der Molen, 1992a, 1992b). The stationarity test of Weber et al. (1992a, 1992b) is derived from evolutionary spectral analysis and is a modification of the test described by Priestley and Subba Rao (1969). Weber et al. (1992a, 1992b) proposed the idea of testing for nonstationarity, and to select (sub-) segments that were classified as stationary for further analyses.

In the present study we attempt to answer the question of whether a spectral analysis method without any assumption of stationarity should be used for psychophysiological heart period variability studies. For this purpose, a quantification is needed of the error that is made by violating the assumption of stationarity when DFT analysis is used for spectral decomposition of heart period power. A typical (stress-reactivity experiment) heart period data set is used to compare power values obtained by the DFT method and power values obtained by the DWT method. Such a comparison is done separately for stationary and nonstationary

segments. We expect that the differences between both methods are greater for nonstationary segments. An ambulatory monitoring device is used to measure the heart period data. We have chosen to employ a within-subjects design with a laboratory psychological stress task followed by a rest period outside of the laboratory. Such a design is expected to produce many stationary as well as nonstationary data segments, even when the data are controlled for physical activity.

Method

Participants

The 40 participants were 15 male and 25 female healthy, first year psychology students from the University of Amsterdam, aged 18–32 years ($M = 21.6$, $SD = 3.2$). All participants received course credits for participation and prizes could be won for the two best performers on the task (Hfl 50 (\$27) and Hfl 25 (\$14)). None of the participants used medication (except contraceptives). Participants refrained from eating, drinking (except water), smoking, and physical exercise 1 hr prior to the commencement of the experiment. Participants were randomly assigned to one of two conditions that were different in the stress task manipulation.

Apparatus

A DOS computer with two external buttons was used for the psychological stress task. The task consisted of an intelligence test with real-time performance feedback combined with a reaction time task. The intelligence test questions were presented on the center of the screen. The reaction time task (used to distract the participant when performing the intelligence test) consisted of randomly timed, falling red and green coins that were presented on both the left and right sides of the screen. Participants were instructed to press the left button when a green coin was falling on the left side, and to press the right button when a green coin was falling on the right side. A combined score of the performance on both tasks was continuously presented to the participant, who was informed about what limit should be reached to be in competition for the prizes.

The physiological measurements were made with the Vrije Universiteit Ambulatory Monitoring System (VU-AMS) version 4.3. This device uses six Ag/AgCl electrodes to record IBIs and thoracic impedance (dZ), and also gives an indication of the amount of physical activity (motility). Details on electrode placement and R-spike detection of this device can be found in de Geus, Willemsen, Klaver, & van Doornen (1995). Cross-instrumental comparison of the VU-AMS with a standard laboratory measurement set-up showed excellent between-subjects and within-subjects correlation of respiration rate, respiratory sinus arrhythmia (RSA), and spectral heart period powers (de Geus et al., 1995).

Procedure

A combined laboratory-ambulatory design was used. Moments of psychological stress in the laboratory were alternated with relaxing moments outside the laboratory. The measurement period always started and ended at 1 p.m. and 4 p.m., respectively. After instruction and informed consent, the ambulatory recording equipment was attached to the participants. Next, they spent approximately 30 min in the laboratory, during which time they were in a seated position in front of the computer. Half of the participants performed the psychological stress task first (which took approximately 15 min), while the other half sat in front of a blank computer screen and waited for 15 min. After this initial period in

the laboratory, participants were brought to another waiting room for a 65-min break, during which time they were able to sit down, relax, and read. Following this break, participants were brought back to the laboratory where all of them performed the psychological stress task. Because half of the participants performed this task for the second time, it consisted of a parallel version of the task used in the first session. Afterwards, the participants were again brought back to the waiting room for a further break of 55 min. During the measurement period, there were also five separate moments when participants were asked to complete a number of questionnaires and then provided saliva samples (results not reported here).

Data Analysis¹

The total interval of recorded physiological data for each participant was nearly 3 hr. These data were analyzed in segments representing 128 s. The segments had an overlap in time of 1 min. This overlap was created because it increased the probability that stationary and nonstationary segments represented the same time periods and experimental situations. Data segments with a measured vertical acceleration motility value above 0.6 gs were discarded from the analyses. This threshold value distinguished (for our VU-AMS device) segments with quiet sitting from segments with walking or turning.

An artifact preprocessing was performed on the IBI data by detecting outlier IBI values (above 1,800 ms or below 300 ms thresholds, and by visual inspection). Because artifacts cannot simply be deleted, because then the continuity of the time is lost, spuriously short IBIs were summed and missing beats were “created” by splitting spuriously long IBIs. In practice, as visual inspection demonstrated, the VU-AMS R-spike detection works very reliably and correction of IBI data appeared to be quite rare.

IBI segments (representing time intervals of 128 s) were selected, from which the IBI mean and variance values were computed.

The stationarity of each IBI data segment was determined by using the stationarity algorithm of Weber et al. (1992a, 1992b). Because this study requires stationary and nonstationary segments to be equal in length, combined with the fact that the data segments already have an overlap in time, the stationary subsegment search algorithm of this test was switched off. A Chi-square value was computed by this test (for each segment), indicative of the degree of nonstationarity (see Weber et al., 1992a). The nominal alpha value was set to .01 to classify a segment as stationary or nonstationary.

Uniformly spaced samples were created by interpolation of the IBI data segments using a Wavelet interpolation algorithm with a quadratic spline as base. Sets of 256 IBIs were first refined in four backwards convolution steps and then low-pass filtered by a Wavelet filter (1 Hz). This procedure resulted in a 16-fold refinement of the IBI data set. In this interpolated IBI data series, new samples were taken at uniformly spaced time intervals (62.5 ms). The first 2,048 newly created samples were taken as a data segment to be further analyzed, corresponding with $2048 * 62.5 \text{ ms} = 128 \text{ s}$. A sample time of 62.5 ms (16 Hz) combined with 2,048 samples was selected to match the frequency bands after DWT with the requested LF and HF heart period variability frequency bands (see below). These values result in segment lengths of 128 s, which is long enough to get a reliable indication of the powers in the LF and

HF bands (see Berntson et al., 1997). Although, as stated by Berntson et al. (1997), reliable estimation of the lowest frequency powers in the ULF band requires a segment length longer than 128 s, the ULF power values were also computed for (exploratory) comparison.

The uniformly spaced data segments were cosine tapered, DFT transformed, smoothed, and the summated powers for the ULF band (below 0.0625 Hz), LF band (0.0625–0.125 Hz), and the HF band (0.125–0.5 Hz) were computed. The small differences between our frequency bands and values regularly used in the literature were necessary to make them comparable with the bands obtained after DWT transformation. Next, the uniformly spaced data segments were Wavelet transformed with the DWT algorithm using a cardinal cubic spline function as base (Chui, 1992, appendix). This base was chosen because it performed in a superior manner to several other orthonormal bases when we attempted to reestablish the exact original signal in the time domain after reconstruction transformation. Frequency-specific time series were created from the DWT transformed data by reconstruction transformation after zeroing all coefficients that represented frequencies outside the requested window. Next, the statistical variance of each frequency-specific time series was derived as an indication of the power within this frequency band. The ULF power was computed as the sum of the variances below 0.0625 Hz, the LF power was computed as the variance of the 0.0625–0.125 Hz window, and the HF power was computed as the sum of the variances of the 0.125–0.25 Hz and 0.25–0.5 Hz windows. Note that the size of a frequency window always doubles after each Wavelet decomposition step. Because the DWT (like DFT) suffers from aliasing effects at both ends, the first and last 40 data points (2.5 s) of the time series were excluded from the derivation of the variances.

The DFT and DWT methods of analysis were compared by computing (for each participant and frequency band) the Pearson product-moment correlation (PMCC) between the log-transformed powers across the segments. Next, the within-subjects mean and standard deviation values were computed (across the segments) for the differences between the log-transformed powers obtained by both methods. These mean and standard deviation values (for each participant and frequency band) were computed separately for stationary and nonstationary segments. Finally, the differences between these stationary and nonstationary mean and standard deviation values were tested (across subjects) with paired *t* tests.

Results

Mental Stress Manipulation

The within-subjects mean heart period during the first session in the lab ($M = 0.79$, $SD = 0.11$) was compared with the mean heart period during the first resting moment ($M = 0.92$, $SD = 0.14$). The mean heart period time was significantly lower during the lab session than during rest, $T(39) = 9.19$, $p < .001$.

Data Example of One Randomly Chosen Participant

Figure 1 shows the IBI data for a nonstationary segment (128 s) of one of the participants during rest. The corresponding power spectrum of these data (computed with the DFT method after creation of uniformly spaced samples) is displayed in Figure 2. Figure 3 shows the same (uniformly spaced) data segment (shown at the top) and seven frequency-specific time series computed from this with the DWT method as described above. Summing these seven time series exactly reproduces the original data (shown at the top) again.

¹The analysis software, which we developed for this study, can be downloaded from www.psy.vu.nl/vu-ams/software/software.ptfap.html.

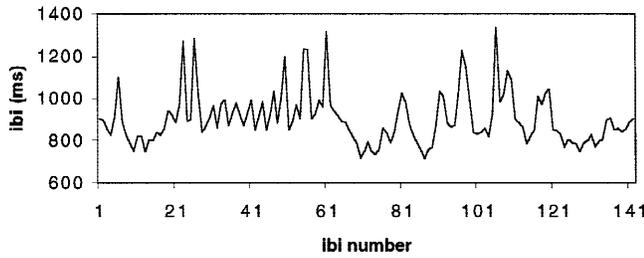


Figure 1. IBI data for a segment classified as nonstationary.

Figure 4A–C shows the scatter plots between log-transformed powers obtained by the DFT and DWT methods for the ULF, LF, and HF bands.

Comparison Between the DWT and DFT Methods of Data Analysis

Correlation values. For each participant, the PMCC correlations between log-transformed power values obtained by the DWT method and log-transformed power values obtained by the DFT method for the ULF, LF, and HF bands were calculated. The mean correlations (across the participants) are presented in Table 1. All these within-subjects correlation values were very high and significant for each participant (all $ps < .01$).

Comparison of the mean values of differences. The mean values (across the participants) of the within-subjects mean values of the differences between the log-transformed DWT and DFT powers are presented in Table 1. Note that these differences were always smaller than 1% of the mean computed power values. One-sample t tests revealed a significant difference of these mean difference values from zero for the ULF band, $t(39) = -17.32$, $p < .001$, for the LF band, $t(39) = 3.72$, $p < .001$, and for the HF band, $t(39) = 17.29$, $p < .001$. Power values computed by DWT were smaller for the ULF band, but larger for the LF and HF bands. Paired t tests were used to test for differences (of the difference between both methods) between stationary and nonstationary segments. These tests revealed no significant differences between stationary and nonstationary mean values for the ULF band, $t(39) = 1.82$, $p = .076$, the LF band, $t(39) = -0.55$, $p = .587$, and the HF band $t(39) = 0.151$, $p = .881$. However, these results do not exclude that the mean standard deviation values of the within-

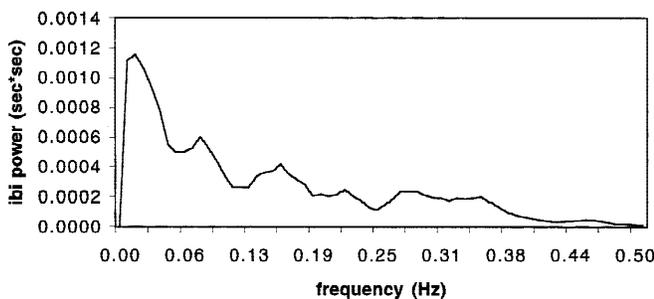


Figure 2. DFT power spectrum of the IBI data shown in Figure 1.

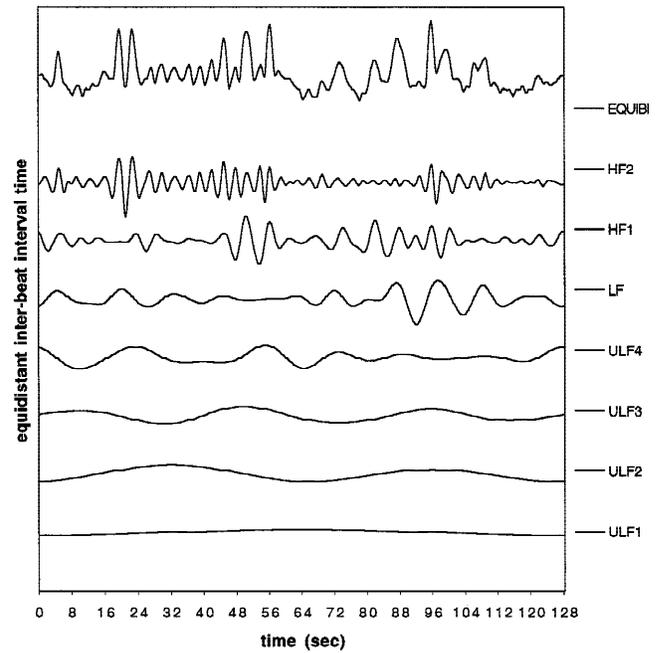


Figure 3. Equidistant sampled time series (top) of the IBI data shown in Figure 1 and the frequency specific time series that were computed from this data using the DWT method.

Table 1. Mean and Standard Deviation Values Across the Participants ($n = 40$) for the Number of Segments, the Within-Subjects Correlations Across the Segments, the Within-Subjects Means Across the segments, the within-subjects mean differences across the Segments, and the Within-Subjects Standard Deviations of the Differences Across the Segments Between the Log-Transformed Power Values Obtained by the DWT Method (WHF,WLF,WULF) and the DFT Method (FHF,FLF,FULF)

	Nonstationary		Stationary	
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
<i>n</i> segments	70.55	18.94	61.95	22.41
$r(\text{WHF},\text{FHF})$.9824	.0225	.9843	.0156
$r(\text{WLF},\text{FLF})$.9575	.0195	.9577	.0204
$r(\text{WULF},\text{FULF})$.9960	.0033	.9951	.0026
Mean((WHF + FHF)/2)	3.08	0.40	3.11	0.45
Mean((WLF + FLF)/2)	2.94	0.29	2.90	0.34
Mean((WULF + FULF)/2)	3.39	0.28	3.25	0.31
Mean(WHF - FHF)	0.0201	0.0073	0.0200	0.0074
Mean(WLF - FLF)	0.0085	0.0132	0.0097	0.0171
Mean(WULF - FULF)	-0.0160	0.0059	-0.0178	0.0063
<i>SD</i> (WHF - FHF)	0.0372	0.0119	0.0315	0.0123
<i>SD</i> (WLF - FLF)	0.0727	0.0122	0.0682	0.0133
<i>SD</i> (WULF - FULF)	0.0275	0.0086	0.0276	0.0067

Note. The percentage values of the mean (W - F) power differences, related to the pooled (W + F)/2 power values, are, respectively, 0.65%, 0.29%, and 0.48% for the nonstationary HF, LF, ULF segments, and 0.65%, 0.33%, and 0.54% for the stationary HF, LF, ULF segments.

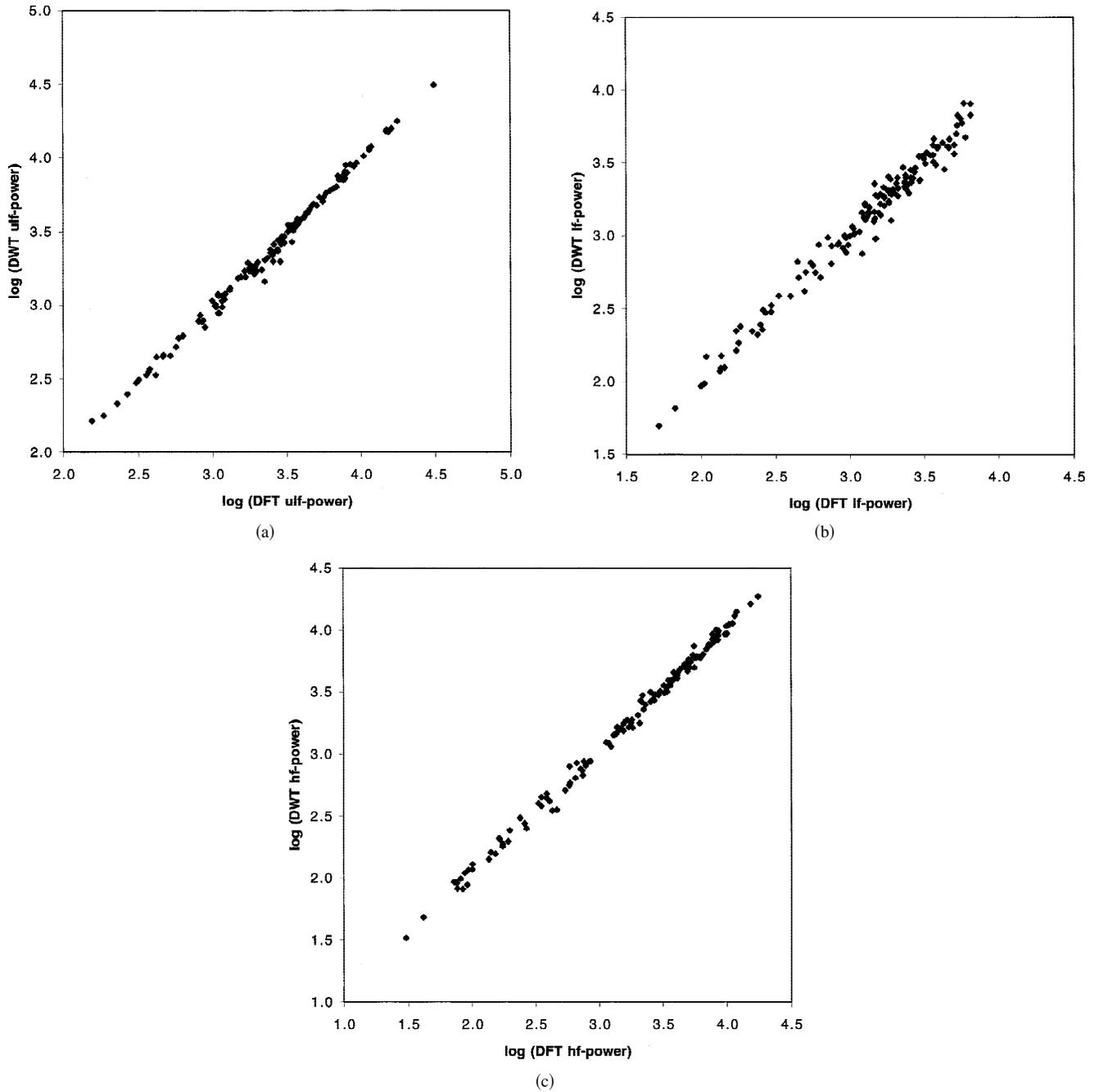


Figure 4. A. Scatter plot for one participant showing the similarity of the ULF powers obtained by the DWT method and the DFT method. B. Scatter plot for one participant showing the similarity of the LF powers obtained by the DWT method and the DFT method. C. Scatter plot for one participant showing the similarity of the HF powers obtained by the DWT method and the DFT method.

subjects differences between both methods may be lower for stationary segments as compared to nonstationary segments.

Comparison of the standard deviation values of differences. The mean values (across the participants) of the within-subjects standard deviation values of the differences between the log-transformed DWT and DFT powers are presented in Table 1. One-sample *t* tests revealed a significant difference of these mean standard deviation values from zero for the ULF band, $t(39) = 22.78, p < .001$, for the LF band, $t(39) = 34.95, p < .001$, and for

the HF band, $t(39) = 17.95, p < .001$. These results are indicative of nonhomogeneous differences between the powers computed by both methods for all bands. Paired *t* tests revealed no significant difference between stationary and nonstationary mean standard deviation values for the ULF band, $t(39) = -0.93, p = .927$. However, a significant difference was found for the LF band, $t(39) = 2.06, p < .05$, and for the HF band, $t(39) = 5.36, p < .001$. For these frequency bands, the mean standard deviation values of the within-subjects differences between both methods were lower for stationary segments (as expected).

Discussion

Inter- and intraindividual comparison demonstrated high mean correlation values ($>.95$) and small mean differences ($<1\%$) between heart period power values obtained by the DFT method and power values obtained by the DWT method. As hypothesized, larger differences between both methods were found for segments that were classified as nonstationary. However, these differences were surprisingly small and more pronounced for the higher frequencies.

For the conditions and settings of this study (a typical stress-reactivity experiment), conclusions based on frequency-specific mean heart period powers using the DFT method (without controlling for stationarity) are probably not very different from conclusions based on the DWT method. Only when very small effects have to be demonstrated (as in the study of Pichot et al., 1999), using DWT can prove beneficial.

The mean heart period interval times were much lower during the sessions in the lab than during rest in the waiting room. It can be concluded from this that the emotional stress manipulation had been successful and that the within-subjects change in heart period over time was large enough to warrant a comparison between both data analysis methods.

The selected motility threshold for rejecting data segments was set at a value (specific for our VU-AMS device) that resulted in the discarding of all periods of walking, turning, and moving. This threshold-level makes our conclusions comparable with most experimental situations where movements are strictly controlled for. It should be noted that the differences between the DWT and DFT methods of spectral analysis might have been larger if the selected motility threshold level was not as strict. Intuitively, segments with more motility are expected to have a higher degree of nonstationarity. This implies that the need for a method of analysis without the assumption of stationarities (like the DWT method) is probably larger when heart period data from ambulatory studies are to be analyzed.

The choice in this study of relatively short data segments (128 s) might have influenced our conclusions as well. Although the lowest frequency powers in the ULF band could not be reliably estimated by the chosen segment length of 128 s (Bertson et al., 1997), both methods produced similar estimations for the mean power values of the ULF band. Longer segments would have produced more reliable estimations of the ULF band powers, but they have a higher risk of failing the stationarity test. Thus, more segments would be classified as nonstationary when the segment length increases. This problem can be resolved by using the method of Weber et al. (1992a, 1992b). This method implies selection of stationary subsegments for computation of the heart period power values. However, a comparison between power values computed from subsegments by the DFT method and power values computed from the original segments by the DWT method results in larger differences, because the selected stationary subsegments of data are no longer representative of the original data (see Grossman, 1992). For longer data segments, an improved solution can be found by using the DWT method applied to the entire nonstationary segments.

Grossman, van Beek, and Wientjes (1990) compared three quantification methods that are in use for estimating RSA. These methods were: (a) the peak-to-valley time-domain method, (b) the Fourier transformation method, and (c) a complex detrending procedure called the moving polynomial method of Porges. They concluded that a very close comparability exists between the three different estimates of RSA and that none of the methods evaluated could claim to produce an obviously superior index of RSA. Our

results are in harmony with this conclusion in that the DWT and DFT methods performed surprisingly similarly for computation of the heart period power values.

We are, nonetheless, convinced that the DWT method, not the DFT, is the theoretically valid method to use with nonstationary data. Heart period data are clearly nonstationary. Hence, it is especially surprising that the DFT method yielded results comparable with the DWT method for heart period data produced by a typical stress-reactivity experiment. The DWT method appears only to be superior for analyzing these data when additional analyses in the time-frequency domain are required.

An example of how this additional information in the time-frequency domain can be used in the interpretation of the DWT transformed data is given by Ivanov et al. (1996). They observed the distribution of the amplitudes of the variations in the beat-to-beat signal corresponding with a specific frequency. This was achieved by fitting specific probability distribution functions to these distribution data and by comparing these fitting parameters between participants.

Another way of using the additional time-frequency information produced by the DWT transformed data (probably more of relevance for RSA) is to observe the patterns of variation in time series together with a time synchronous time series of another signal. When the respiration and heart period signals are both DFT transformed, the coherence can be computed as a measure of the transfer from respiration to heart rate. Although this technique works for stationary data, it probably does not result in a stable coherency spectrum when the data are nonstationary. However, the coupling between two signals for specific frequency intervals can also be estimated by the DWT method. For this method, both signals should first be transformed to several time series by Wavelet transformation (each representing a certain frequency window as described above). Next, an impression of the (nonstationary) transfer can be obtained simply by calculating the square of the maximal cross-correlation for each (frequency-window specific) set of time series. The lag of the maximal cross-correlation gives an impression of the phase shift between the respiration and heart period signals for that specific frequency window. Thus, the additional time-frequency information produced by the DWT method can be used for obtaining a nonstationary index of coupling between heart period data and respiration.

Finally, DWT transformation can also be used for data reduction or smoothing, by respectively selecting coefficients above a certain quantile threshold or reducing the amplitude of coefficients representing unwanted frequencies. Wavelet data reduction or smoothing is able to eliminate rapid oscillations in the signal, but without eliminating sharp turns with a lower base frequency. In general, Wavelet transformation can be used as a filter by making all coefficients representing a certain frequency window equal to zero (or by reducing the amplitudes) before inverse transformation. This filter has an advantage over using the Fourier transform for filtering because no information from the signal in the time domain is lost (there is no assumption of stationarity).

In summary, it can be concluded that for a typical stress-reactivity experiment, mean heart period powers computed by the DFT method (without controlling for stationarity) are only marginally different from mean powers computed by the DWT method. Differences between these methods are partially the result of the error that is made by violating the assumption of stationarity when the DFT method is used. Because the differences are surprisingly small, the DWT method is only superior for analyzing heart period data when additional analyses in the time-frequency domain are required.

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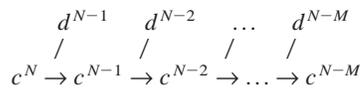
(RECEIVED April 22, 1999; ACCEPTED August 25, 2000)

APPENDIX: THE DWT FORMULAS

The DWT decomposition algorithm starts at the largest hierarchy and works towards the smallest (from Chui, 1992, 5.4.48):

$$c_k^{j-1} = \sum_l a_{l-2k} c_l^j$$

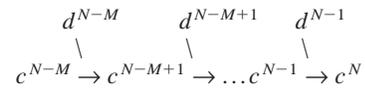
$$d_k^{j-1} = \sum_l b_{l-2k} c_l^j$$



a_k, b_k = wavelet decomposition sequences

The DWT reconstruction algorithm starts at the smallest hierarchy and works towards the largest (from Chui, 1992, 5.4.49):

$$c_k^j = \sum_l [p_{k-2l} c_l^{j-1} + q_{k-2l} d_l^{j-1}]$$



p_k, q_k = wavelet reconstruction sequences